Lessons 23 and 24: Surface Integrals

July 28, 2016

1. Evaluate $\int\int_S 6xy\,dS$ where S is the portion of the plane $x + y + z = 1$ that lies in the 1st octant.
   Answer: $\frac{\sqrt{3}}{4}$

2. Let S be the sphere $x^2 + y^2 + z^2 = 4$ with positive (outward) orientation. Let $\mathbf{F}(x, y, z) = \langle x, y, z\rangle$. Calculate the flux of F across S (i.e. calculate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$).
   Answer: 32π

3. Evaluate $\int\int_S z\,dS$, where S is the upper hemisphere of a sphere with radius 3.
   Answer: 27π

4. Evaluate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle z, y, x\rangle$, and S is the helicoid $x = u\cos v$, $y = u\sin v$, $z = v$, $0 \le u \le 1$, $0 \le v \le \pi$ with downward orientation.
   Answer: $-\pi$

5. Evaluate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle xy, 2yz, xyz\rangle$ and S is the part of the paraboloid $z = x^2 + y^2$ lying above $z = 0$ and below $z = 4$ with upward orientation.
   Answer: $\frac{-128\pi}{3}$

6. Evaluate $\int\int_S y + z\,dS$, where S is the portion of the plane $z = 4 - y$ lying above the circle $x^2 + y^2 = 3$.
   Answer: $12\sqrt{2}\pi$